Partial Fraction Decomposition

When the zeros (denoted p_i) of $Q(z)$, which are also the poles of $X(z)$, are of order 1, we have:

$$\frac{P_0(z)}{Q(z)} = \sum_{i=1}^{N} \frac{\alpha_i}{z - p_i}, \quad \alpha_i = \left[\frac{P_0(z)}{Q(z)}(z - p_i)\right]_{z=p_i}$$

If a zero (denoted p_n) of $Q(z)$ is of order $q > 1$, the decomposition becomes:

$$\frac{P_0(z)}{Q(z)} = \sum_{\substack{i=1 \\ i \neq n}}^{N} \frac{\alpha_i}{z - p_i} \bullet \sum_{j=1}^{q} \frac{\beta_j}{(z - p_n)^j}$$

with

$$\beta_j = \frac{1}{(q-j)!}\left[\frac{d^{q-j}}{dz^{q-j}}\left((z - p_n)^q \frac{P_0(z)}{Q(z)}\right)\right]_{z=p_n}$$